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Non-minimal Higgs content in standard-like models from D-branes at a $\mathbb{Z}_N$ singularity

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ABSTRACT

We show that attempts to construct the standard model, or the MSSM, by placing D3-branes and D7-branes at a $\mathbb{Z}_N$ orbifold or orientifold singularity all require that the electroweak Higgs content is non-minimal. For the orbifold the lower bound on the number $n(H)+n(\bar{H})$ of electroweak Higgs doublets is the number $n(q^c_L)=6$ of quark singlets, and for the orientifold the lower bound can be one less. As a consequence there is a generic flavour changing neutral current problem in such models.

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The construction of models that lead to something like the standard model has been a major activity in string theory for many years. (See, for example, [1] for a review.) The D-brane world offers an attractive, bottom-up route to getting standard-like models from Type II string theory that has been particularly popular recently. Open strings that begin and end on a stack of M D-branes generate the gauge bosons of the group $U(M)$ living in the world volume of the D-branes. Recently "intersecting brane" models have enjoyed considerable popularity. (See [2] for a recent review.) In these models one starts with one stack of 3 D-branes, another of 2, and n other stacks each having just 1 D-brane, thereby generating the gauge group $U(3) \otimes U(2) \otimes U(1)^n$. The D4-,5- or 6-branes wrap the three large spatial dimensions and respectively 1-, 2- or 3-cycles of the six-dimensional internal space (typically a torus T^6 or a Calabi-Yau 3-fold) on which the theory is compactified. Then fermions in bi-fundamental representations of the corresponding gauge groups can arise at the multiple intersections of such stacks [3], but to get $D = 4$ *chiral* fermions the intersecting branes should sit at a singular point in the space transverse to the branes, an orbifold fixed point, for example. In general, such configurations yield a non-supersymmetric spectrum, so that, to avoid the hierarchy problem, the string scale associated with such models must be no more than a few TeV. If so, some of the compactified dimensions must be large, and this raises the question of how the required high Planck energy scale associated with gravitation emerges. Provided that the volume of the space orthogonal to the wrapped space is sufficiently large, it seems that both scales can be accommodated [4, 5]. However, a generic feature of these models is that flavour changing neutral currents are generated by four-fermion operators induced by string instantons [6]. Although such operators allow the emergence of a realistic pattern of fermion masses and mixing angles, the severe experimental limits on flavour changing neutral currents require that the string scale is rather high, of order 10^4 TeV. This makes the fine tuning problem very severe and renders the viability of these models highly questionable. In a non-supersymmetric theory the cancellation of the Ramond-Ramond (RR) tadpoles does *not* ensure the cancellation of the Neveu-Schwarz-Neveu-Schwarz (NSNS) tadpoles. NSNS tadpoles are simply the first derivatives of the scalar potential with repect to the scalar fields, specifically the complex structure moduli and the dilaton. Thus a consequence of the non-cancellation is that there is an instability in the complex structure moduli [7]. One way to stabilise the complex structure moduli is to use an orbifold, rather than a torus, for the space wrapped by the D-branes. If the embedding is supersymmetric, RR tadpole cancellation ensures the cancellation of the NSNS tadpoles too [8, 9]. Using D6-branes and a $\mathbb{Z}_4, \mathbb{Z}_4 \times \mathbb{Z}_2$ or $\mathbb{Z}_6$ orientifold, some or all of the complex structure moduli may be stabilised [10, 11, 12]. Although a semi-realistic three-generation model has been obtained this way [12], it has non-minimal Higgs content, so it too will have flavour changing neutral currents.

For all of these reasons it seems timely to re-examine the viability of the earlier "bottom-up" models [13, 14, 15, 16], the study of which began before the intersecting brane models became popular. In these one starts with a set of $n + 5$ D3-branes situated at an orbifold $T^6/\mathbb{Z}_N$ singularity. In contrast to the intersecting

brane models discussed above, the gauge group $U(3) \otimes U(2) \otimes U(1)^n$ is obtained by choosing a suitable embedding $\gamma_{\theta,3}$ of the action of the generator θ of the point group $\mathbb{Z}_N$ on the Chan-Paton indices of the D3-branes. Before the orbifold projection, a set of $n+5$ D3-branes generically gives an $\mathcal{N}=4$ supersymmetric $U(n+5)$ gauge theory. In $\mathcal{N}=1$ language this consists of an adjoint vector multiplet and 3 adjoint chiral multiplets. In terms of component fields there are $U(n+5)$ gauge fields, four adjoint fermions, transforming as the $\mathbf{4}$ representaion of $SU(4)$, and six adjoint scalars, transforming as the $\mathbf{6}$ of $SU(4)$. The action of θ on the four fermions is given by

$$R_4(\theta) = \mathrm{diag}\left(e^{2\pi i a_1/N}, e^{2\pi i a_2/N}, e^{2\pi i a_3/N}, e^{2\pi i a_4/N}\right) \tag{1}$$

where the a_α are integers satisfying

$$a_1 + a_2 + a_3 + a_4 = 0 \bmod N \tag{2}$$

The action of the point group on the six scalars is given by

$$R_6(\theta) = \mathrm{diag}\left(e^{2\pi i b_1/N}, e^{-2\pi i b_1/N}, e^{2\pi i b_2/N}, e^{-2\pi i b_2/N}, e^{2\pi i b_3/N}, e^{-2\pi i b_3/N}\right) \tag{3}$$

with

$$b_1 = a_2 + a_3, \quad b_2 = a_3 + a_1, \quad \text{and} \quad b_3 = a_1 + a_2 \tag{4}$$

In general, $\gamma_{\theta,3}$ is an $(n+5)\times(n+5)$ matrix satisfying

$$\gamma_{\theta,3}^N = \pm 1 \tag{5}$$

In a Cartan-Weyl basis it can be written in the form

$$\gamma_{\theta,3} = e^{-2\pi i V_3 . H} \tag{6}$$

with H_I the Cartan generators of $U(n+5)$. Depending on the sign choice in (5), the vector V_3 has the form

$$\begin{aligned} V_3 &= \frac{1}{N}(0^{n_0}, 1^{n_1}, \ldots, j^{n_j}, \ldots) && (7)\\ \text{or } V_3 &= \frac{1}{N}\left(\left(\frac{1}{2}\right)^{n_1}, \left(\frac{3}{2}\right)^{n_3}, \ldots, \left(j+\frac{1}{2}\right)^{n_{2j+1}}, \ldots\right) && (8) \end{aligned}$$

where $\sum_j n_j = n+5$ is the total number of D3-branes. Here we are using the shorthand notation j^{n_j} to denote n_j entries j. In what follows we shall take $\gamma_{\theta,3}^N = +1$. However, a similar analysis can be made for the case $\gamma_{\theta,3}^N = -1$ with identical conclusions.

Gauge bosons arise from open strings that begin on a D3-brane and end on a D3-brane, i.e. they are (33)-sector states. The gauge bosons that survive the orbifold projection (i.e. are point-group invariant) are those associated with the (neutral)

Cartan-Weyl generators, plus those associated with the charged generators having root vectors $\rho_3 = (\underline{1, -1, 0^{n+3}})$ that satisfy

$$\rho_3.V_3 = 0 \bmod 1 \tag{9}$$

(The underlining signifies that all permutations of the underlined entries are to be included.) Then (7) gives the surviving D3-brane gauge group as $\bigotimes_j U(n_j)$. In particular, we choose V_3 to have the form

$$V_3 = \frac{1}{N}(x^3, y^2, d_1, d_2, ..., d_i, ...) \tag{10}$$

where x and y are two different integers, and the single distinct entries d_i also differ from x and y, so that

$$\forall i \ x \neq y \neq d_i \neq x \bmod N \tag{11}$$

Then $n_x = 3$, $n_y = 2$ and $n_j = 1$ if $j \neq x, y$, and the D3-brane gauge group is $U(3) \otimes U(2) \bigotimes_i U(1)_i$.

Point group invariance also implies that there is surviving matter on the D3-branes with root vectors ρ_3 of the above form that satisfy [13, 16]

$$\begin{aligned} \rho_3.V_3 &= -\frac{a_\alpha}{N} \bmod 1 \quad \text{(fermions)} &(12)\\ &= \frac{b_r}{N} \bmod 1 \quad \text{(scalars)} &(13) \end{aligned}$$

where a_α $(\alpha = 1, 2, 3, 4)$ are defined in (1) and (2), and b_r $(r = 1, 2, 3)$ are defined in (4). This gives rise matter in bi-fundamental representations of the D3-brane gauge group:

$$\sum_{\alpha=1}^{4}\sum_{j=0}^{N-1}(\mathbf{n}_j, \bar{\mathbf{n}}_{j+a_\alpha}) \quad \text{(fermions)} \tag{14}$$

$$\sum_{r=1}^{3}\sum_{j=0}^{N-1}(\mathbf{n}_j, \bar{\mathbf{n}}_{j-b_r}) \quad \text{(scalars)} \tag{15}$$

where all sub-indices are understood modulo N, and the fundamental $(\mathbf{n}_j)$ (anti-fundamental $(\bar{\mathbf{n}}_j)$) representation of $SU(n_j)$ has respectively $+1(-1)$ units of the charge Q_j associated with the $U(1)$ factor in $U(n_j) = U(1) \otimes SU(n_j)$.

The obvious way to get three generations of quark doublets Q_L each transforming as the **3** of $SU(3)_c$ and the **2** of $SU(2)_L$ is to use the embedding (10) of θ and to choose

$$(a_1, a_2, a_3, a_4) \equiv (a, a, a, c = -3a) \ (\mathrm{mod} N) \tag{16}$$

Then, taking $y = x + a$, the $j = x$ contribution to the sum (14) gives precisely the required three copies of the representation $(\mathbf{3}, \bar{\mathbf{2}})$ of $SU(3)_c \otimes U(2)_L$. Since these are the only **3** representations in the standard model, we require also that

$$\forall i \quad x + a \neq d_i \neq x - 3a \ (\mathrm{mod} N) \tag{17}$$

The contributions to the sum (14) from the terms with $j = x - a_\alpha$ give fermions transforming as $\bar{\mathbf{3}}$ representations of $SU(3)_c$:

$$3(\mathbf{n}_{x-a}, \bar{\mathbf{3}}) + (\mathbf{n}_{x+3a}, \bar{\mathbf{3}}) \tag{18}$$

Provided that they have the correct weak hypercharges, these are potentially quark singlet states u_L^c and d_L^c and the corresponding states in other generations. However, to ensure the absence of unwanted $(\mathbf{2}, \bar{\mathbf{3}})$ states we require that $x - a \neq y \neq x + 3a$ (mod$N$), i.e. that

$$2a \neq 0 \bmod N \tag{19}$$

The states (18) will appear in the fermionic spectrum as $(\bar{\mathbf{3}}, \mathbf{1})$ representations of $SU(3)_c \otimes SU(2)_L$ if $x - a$ and/or $x + 3a$ is one of the entries d_i in V_3 given in (10). The number $n^{(33)}(\bar{\mathbf{3}}, \mathbf{1})$ of such states arising in the (33) sector is given by

$$n^{(33)}_{\text{fermions}}(\bar{\mathbf{3}}, \mathbf{1}) = \sum_i (3\delta_{d_i, x-a} + \delta_{d_i, x+3a}) \tag{20}$$

The $j = y$ and $j = y + b_r$ contributions to the scalar spectrum (15) give states transforming respectively as $\mathbf{2}$ and $\bar{\mathbf{2}}$ of $U(2)$. With the values (16) for the a_α we see from (4) that

$$(b_1, b_2, b_3) = (2a, 2a, 2a) \tag{21}$$

so the doublet states are

$$3(\mathbf{2}, \bar{\mathbf{n}}_{y-2a}) + 3(\mathbf{n}_{y+2a}, \bar{\mathbf{2}}) \tag{22}$$

These are potentially Higgs doublets, and since $y = x + a$ we find that the numbers $n^{(33)}(\mathbf{1}, \mathbf{2})$ and $n^{(33)}(\mathbf{1}, \bar{\mathbf{2}})$ of $(\mathbf{1}, \mathbf{2})$ and $(\mathbf{1}, \bar{\mathbf{2}})$ representations of $SU(3)_c \otimes U(2)_L$ in the (33) sector are

$$n^{(33)}_{\text{scalars}}(\mathbf{1}, \mathbf{2}) = 3 \sum_i \delta_{d_i, x-a} \tag{23}$$

$$n^{(33)}_{\text{scalars}}(\mathbf{1}, \bar{\mathbf{2}}) = 3 \sum_i \delta_{d_i, x+3a} \tag{24}$$

The weak hypercharge Y is some *a priori* unknown linear combination of the $U(1)$ charges from each factor in the gauge group:

$$Y = \alpha_x Q_x + \alpha_y Q_y + \sum_i \alpha_i Q_{d_i} + ... \tag{25}$$

where the ... represents contributions from the D7-brane gauge groups, which will be discussed below. To get the correct weak hypercharges for the 3 quark doublets Q_L, for the u_L^c, d_L^c quark singlet states and the corresponding states in other generations, as well as for the electroweak Higgs doublets that arise in the (33)-sector, we take

$$Y = \frac{1}{6} Q_x + \frac{1}{2} \sum_{i \in I_d} Q_{d_i} - \frac{1}{2} \sum_{i \in I_u} Q_{d_i} + ... \tag{26}$$

where the sets I_d and I_u are non-overlapping and together include all of the d_i. We are dropping a contribution proportional to $Q_x + Q_y + \sum_i Q_{d_i}$ which is zero for all states. Clearly the (33) sector yields at least as many Higgs doublets as quark singlet q_L^c states.

The (33)-sector fermion spectrum generally makes the non-abelian gauge symmetries $SU(n_j)$ anomalous, and indeed in our model the colour-triplet ($\mathbf{3}$ and $\bar{\mathbf{3}}$) fermions do make $SU(3)_c$ anomalous. This reflects the fact that a general collection of D3-branes has uncancelled RR tadpoles. The required twisted tadpole cancellation is effected locally by the introduction of D7_r-branes at the orbifold fixed point at which the D3-branes are located. The D7_r-branes wrap the three large spatial dimensions and the four compact dimensions perpendicular to the rth complex plane, where $r = 1, 2, 3$. As in (6), the action of the $\mathbb{Z}_N$ point group on the Chan-Paton indices is encoded in a u^r-component vector of the form

$$V_{7_r} = \frac{1}{N}(0^{u_0^r}, 1^{u_1^r}, ..., j^{u_j^r}, ...) \tag{27}$$

$$\text{or } V_{7_r} = \frac{1}{N}\left(\left(\frac{1}{2}\right)^{u_1^r}, \left(\frac{3}{2}\right)^{u_3^r}, ..., \left(j+\frac{1}{2}\right)^{u_{2j+1}^r}, ...\right) \tag{28}$$

with $\sum_j u_j^r \equiv u^r$; (27) (or (28)) applies when $\gamma_{\theta,7_r}^N = +1$ or (-1). The D7_r-brane gauge group is $\bigotimes_j U(u_j^r)$. The introduction of D7_r-branes leads also to (37_r)- and (7_r3)-sector states which arise from open strings with one end on a D3-brane and the other on a D7_r-brane. An advantage of the bottom-up models is that, since all of the matter is located at the (same) orbifold fixed point, these models are free of the instanton-induced flavour changing neutral current problem that afflicts the intersecting brane models. In the (37_r)- and (7_r3)-sectors the point group generator is represented by the $(5 + n + u^r)$-component vector

$$V_{37_r} \equiv (V_3; V_{7_r}) \tag{29}$$

The (37_r)- and (7_r3)-states are described by weight vectors ρ_{37_r} of the form

$$\rho_{37_r} = \left(\underline{\pm 1, 0^{n+4}}; \underline{\mp 1, 0^{u^r-1}}\right) \tag{30}$$

In general the surviving states satisfy [13, 16]

$$\rho_{37_r}.V_{37_r} = -\frac{b_r}{2N} \bmod 1 \quad \text{(fermions)} \tag{31}$$

$$= \frac{b_s + b_t}{2N} \bmod 1 \quad r \neq s \neq t \neq r \quad \text{(bosons)} \tag{32}$$

For the three-generation model we are considering, the twists $b_r = 2a$ given in (21) are all even, in which case (27) applies when $\gamma_{\theta,3}^N = +1$, as we have assumed. The (37_r)+(7_r3)-sector spectrum is again given in terms of bi-fundamental representations,

but now of the D3-brane and D7$_r$-brane gauge groups:

$$\sum_{j=0}^{N}\Big[(\mathbf{n}_j,\bar{\mathbf{u}}^r_{j+\frac{1}{2}b_r})+(\bar{\mathbf{n}}_j,\mathbf{u}^r_{j-\frac{1}{2}b_r})\Big]\quad\text{(fermions)}\tag{33}$$

$$\sum_{j=0}^{N}\Big[(\mathbf{n}_j,\bar{\mathbf{u}}^r_{j-\frac{1}{2}(b_s+b_t)})+(\bar{\mathbf{n}}_j,\mathbf{u}^r_{j+\frac{1}{2}(b_s+b_t)})\Big]\quad\text{(scalars)}\tag{34}$$

The contribution to (33) from the $j=x$ term gives fermions transforming as $\mathbf{3}$ and $\bar{\mathbf{3}}$ representations of $SU(3)_c$. To avoid unwanted $\mathbf{3}$ states we must therefore require that $u^r_{x+a}=0$ for all $r=1,2,3$. The other term gives a number $n^{(37)}(\bar{\mathbf{3}},\mathbf{1})$ of states transforming as the $(\bar{\mathbf{3}},\mathbf{1})$ representation of $SU(3)_c\otimes SU(2)_L$:

$$n^{(37)}_{\text{fermions}}(\bar{\mathbf{3}},\mathbf{1})=\sum_r u^r_{x-a}\tag{35}$$

These too are potentially quark singlet states u^c_L, d^c_L and the corresponding states in other generations, provided that they have the correct weak hypercharges. Similarly, the $j=y=x+a$ term in (34) gives scalar states transforming as $(\mathbf{1},\mathbf{2})$ and $(\mathbf{1},\bar{\mathbf{2}})$ representations of $SU(3)_c\otimes U(2)_L$:

$$n^{(37)}_{\text{scalars}}(\mathbf{1},\mathbf{2})=\sum_r u^r_{x-a}\tag{36}$$

$$n^{(37)}_{\text{scalars}}(\mathbf{1},\bar{\mathbf{2}})=\sum_r u^r_{x+3a}\tag{37}$$

and these are potentially electroweak Higgs doublets, provided that they have the correct weak hypercharges. The correct hypercharges for both the quark singlet states and the electroweak Higgs doublets is achieved by including in (25) the contributions from the $U(1)$ charges Q_{j^r} of the $U(u^r_j)$ factors in the D7-brane gauge group. We find that the (37)-sector states have the correct standard-model values if we take

$$Y=\frac{1}{6}Q_x+\frac{1}{2}\sum_{i\in I_d}Q_{d_i}-\frac{1}{2}\sum_{i\in I_u}Q_{d_i}+\frac{1}{2}\sum_{r,j^r\in J^r_d}Q_{j^r}-\frac{1}{2}\sum_{r,j^r\in J^r_u}Q_{j^r}\tag{38}$$

where for each r the sets J^r_d and J^r_u are non-overlapping and include all values of j^r. Thus, like the (33)-sector, the (37)-sector too yields at least as many Higgs doublets as there are quark singlet q^c_L states. Consequently, the total number $n(q^c_L)$ of quark singlet states is not greater than the number $n(H)+n(\bar{H})$ of Higgs doublets:

$$n(q^c_L)\leq n(H)+n(\bar{H})\tag{39}$$

Since any standard-like model must have three u^c_L and three d^c_L states, so that $n(q^c_L)=6$, it follows that

$$n(H)+n(\bar{H})\geq 6\tag{40}$$

and that the Higgs content cannot be that of the standard model, nor of its minimal supersymmetric extension, the MSSM.

In any case, it is well known that such orbifold models cannot have a standard-like fermionic spectrum. Twisted tadpole cancellation implies the cancellation of the non-abelian anomalies [13, 17, 18, 19]. Thus, after the inclusion of the (37_r)+(7_r3)-sector matter, the same number of fermions in fundamental and anti-fundamental representations of $SU(n_j)$ must be present for each $SU(n_j)$. In other words the net charge Q_j carried by fermions must vanish for every j. However, having 3 copies of Q_L transforming as the $(\mathbf{3},\bar{\mathbf{2}})$ representation of $SU(3)_c \otimes U(2)_L$ generates 9 copies of the $\bar{\mathbf{2}}$ representation of $U(2)_L$ having a total $Q_y = -9$, so to cancel it the remaining fermions must include (at least) 9 copies of the $\mathbf{2}$ representation of $U(2)_L$ having a total $Q_y = 9$. Besides the 3 quark doublets, the standard model, of course, has precisely 3 copies of the $\mathbf{2}$ representation of $SU(2)_L$, corresponding to the 3 lepton doublets L. Thus this method of generating the quark doublets inevitably entails the existence of 6 vector-like lepton doublets $L + \bar{L}$, not present in the standard model, and the first bottom-up attempts [13, 14] to get the standard model indeed suffered from this defect.

The only escape is to arrange that three quark doublets do not all have the same Q_y charge [20]. We require $2(\mathbf{3},\bar{\mathbf{2}}) + (\mathbf{3},\mathbf{2})$ representations of $SU(3)_c \otimes U(2)_L$ (or equivalently $(\mathbf{3},\bar{\mathbf{2}}) + 2(\mathbf{3},\mathbf{2})$) which have a total $Q_y = -3$ (or $Q_y = +3$). This can be cancelled by 3 lepton doublets transforming respectively as the $\mathbf{2}$ (or $\bar{\mathbf{2}}$) representation of $U(2)_L$. Getting both representations is possible only when the $\mathbb{Z}_N$ orbifold singularity at which the D3-branes are situated is on an orientifold plane [20, 16]. In the orientifolds that we are considering the point group quotienting the torus T^6 is enlarged. It is generated by $\{\theta, \Omega\}$ where, as before, θ is the generator of $\mathbb{Z}_N$ and the new generator Ω is the world-sheet parity operator. Thus the complete orientifold group is $\mathbb{Z}_N + \Omega\mathbb{Z}_N$. The requirement of invariance under the action of the extra generator Ω restricts the form of the embedding $\gamma_{\theta,3}$ given in (6), and hence the form of V_3 given in (7,8). Ω acts on the diagonal matrix $\gamma_{\theta,3}$ as complex conjugation, and (for the case that $\gamma_{\theta,3}^N = +1$) the invariance requires [21] that V_3 has the form

$$V_3 = (\tilde{V_3}; -\tilde{V_3}) \tag{41}$$

and $\tilde{V_3}$ is given by

$$\tilde{V_3} = \frac{1}{N}(0^{n_0}, 1^{n_1}, ..., P^{n_P}) \tag{42}$$

with $P \equiv \left[\frac{N}{2}\right]$ the largest integer not greater than $\frac{N}{2}$. Then the gauge group factor $U(n_j)$ is the same as $U(n_{-j}) \equiv U(n_{N-j})$ and the two are exchanged under the action of Ω. Invariance requires that they are identified. The (33)-sector spectrum may be calculated as before, using equations (9, 12, 13), but with V_3 replaced by $\tilde{V_3}$ and now with

$$\rho_3 = (\underline{\pm 1, \pm 1, 0, 0, ...}) \tag{43}$$

and all four combinations of signs allowed. For a general embedding the D3-brane gauge group is $SO(2n_0) \bigotimes_{j=1}^{P} U(n_j)$. The extra states included in (43) lead to extra

fermions and scalars surviving the projection. Instead of (14) and (15), we now have

$$\sum_{\alpha=1}^{4}\sum_{j=0}^{N-1}[(\mathbf{n}_j,\bar{\mathbf{n}}_{j+a_\alpha})+(\mathbf{n}_j,\mathbf{n}_{-j-a_\alpha})+(\bar{\mathbf{n}}_j,\bar{\mathbf{n}}_{-j+a_\alpha})+ \\ +(\mathbf{n}_j\times\mathbf{n}_j)_a\delta_{2j,-a_\alpha}+(\bar{\mathbf{n}}_j\times\bar{\mathbf{n}}_j)_a\delta_{2j,a_\alpha}] \quad \text{(fermions)} \tag{44}$$

$$\sum_{r=1}^{3}\sum_{j=0}^{N-1}[(\mathbf{n}_j,\bar{\mathbf{n}}_{j-b_r})+(\mathbf{n}_j,\mathbf{n}_{-j+b_r})+(\bar{\mathbf{n}}_j,\bar{\mathbf{n}}_{-j-b_r})+ \\ +(\mathbf{n}_j\times\mathbf{n}_j)_a\delta_{2j,b_r}+(\bar{\mathbf{n}}_j\times\bar{\mathbf{n}}_j)_a\delta_{2j,-b_r}] \quad \text{(scalars)} \tag{45}$$

Here $(\mathbf{n}_j\times\mathbf{n}_j)_a$ is the antisymmetric $\frac{1}{2}n_j(n_j-1)$-dimensional representation of $U(n_j)$ that arises in the product $\mathbf{n}_j\times\mathbf{n}_j$, and similarly for $(\bar{\mathbf{n}}_j\times\bar{\mathbf{n}}_j)_a$.

When $\tilde{V_3}$ has the form given in (10, 11), namely

$$\tilde{V_3}=\frac{1}{N}(x^3,y^2,d_1,d_2,...,d_i,...) \tag{46}$$

with x and y non-zero and

$$x\neq y\neq d_i\neq x \bmod N\ (\forall i) \tag{47}$$

then, as before, the gauge group is $U(3)\otimes U(2)\bigotimes_i U(1)_i$. Also as before, quark doublets Q_L transforming as the $(\mathbf{3},\bar{\mathbf{2}})$ representation of $SU(3)_c\otimes U(2)_L$ can only arise from the $j=x$ contribution to the first term of (44), and to get 2 copies two of the a_α must be equal. So we take

$$\begin{aligned}(a_1,a_2,a_3,a_4) &= (a,a,b,c=-(2a+b))\ (\mathrm{mod}N) && (48)\\ (b_1,b_2,b_3) &= (a+b,a+b,2a)\ (\mathrm{mod}N) && (49)\end{aligned}$$

with

$$a\neq b\neq c\neq a\ (\mathrm{mod}N) \tag{50}$$

so as to avoid the occurrence of three identical quark doublets. Then, the required 2 copies of Q_L arise if $y=x+a$. The $(\mathbf{3},\mathbf{2})$ fermion representation of $SU(3)_c\otimes U(2)_L$ arises from the $j=x$ contribution to the second term of (44), and we get the required single copy if $y=-x-b$. The definition (38) of the weak hypercharge again ensures that all of these states have the correct standard-model weak hypercharge $Y=\frac{1}{6}$. (We also get the required $(\mathbf{3},\mathbf{2})$ fermion representation if $y=-x-c$, but the physics in the two cases is, of course, identical.) Note that in the orientifold case the contribution to Y proportional to $Q_x+Q_y+\sum_i Q_{d_i}$ is no longer automatically zero for all states. However, since we require that both the $(\mathbf{3},\bar{\mathbf{2}})$ and the $(\mathbf{3},\mathbf{2})$ fermion states have the same weak hypercharge, this contribution must be absent. Thus $\tilde{V_3}$ has the form:

$$\tilde{V_3}=\frac{1}{N}\left[\left(-\frac{a+b}{2}\right)^3,\left(\frac{a-b}{2}\right)^2,d_1,d_2,...\right] \tag{51}$$

with all of the entries non-zero and unequal in order to avoid enlargement of the gauge group. For the same reason we also require that

$$
\begin{aligned}
-\frac{1}{2}(a+b) &\neq +\frac{1}{2}(a+b),\ \pm\frac{1}{2}(a-b),\ \pm d_i \quad (\mathrm{mod} N) && (52)\\
-\frac{1}{2}(a-b) &\neq +\frac{1}{2}(a-b),\ \pm d_i \quad (\mathrm{mod} N) && (53)\\
-d_i &\neq d_i,\ \pm d_j\ (j\neq i) \quad (\mathrm{mod} N) && (54)
\end{aligned}
$$

To ensure that there is no unwanted vector-like quark doublet matter, we require further that

$$2a \neq 0, \quad 2b \neq 0\ (\mathrm{mod} N) \tag{55}$$

Since we are assuming that $\gamma_{\theta,3}^N = +1$, as before, then $a-b = 0 \bmod 2 = a+b$. Quark singlet states $\bar{q}_L^c$ transforming as the $(\mathbf{3},\mathbf{1})$ representation and q_L^c states transforming as the $(\bar{\mathbf{3}},\mathbf{1})$ representation of $SU(3)_c \otimes SU(2)_L$ can also arise from the $j=x$ and $j = x - a_\alpha$ contributions to (44). Proceeding as before, we find that the number of such states is given by

$$
\begin{aligned}
n^{(33)}_{\mathrm{fermions}}(\mathbf{3},\mathbf{1}) &= 2\delta_{2a+b,0} + \delta_{a+2b,0} + \sum_i \left[\delta_{d_i,\frac{1}{2}(5a+3b)} + \delta_{d_i,-\frac{1}{2}(5a+3b)}\right] && (56)\\
n^{(33)}_{\mathrm{fermions}}(\bar{\mathbf{3}},\mathbf{1}) &= \delta_{3a+2b,0} + 3\sum_i(\delta_{d_i,\frac{1}{2}(3a+b)} + \delta_{d_i,-\frac{1}{2}(3a+b)}) + \\
&\quad + \sum_i(\delta_{d_i,\frac{1}{2}(a+3b)} + \delta_{d_i,-\frac{1}{2}(a+3b)}) && (57)
\end{aligned}
$$

Again, (38) ensures that these states have the correct standard-model weak hypercharge assignments. To exclude the unwanted $\bar{q}_L^c$ states we must take

$$2a+b \neq 0 \neq a+2b \text{ and } \forall i\ \pm d_i \neq \frac{1}{2}(5a+3b)\ (\mathrm{mod} N) \tag{58}$$

The embedding (48), and the spectrum, is supersymmetric in the special case that $c = 0 = 2a+b \bmod N$. Thus the first of these inequalities excludes the supersymmetric embedding, and hence the possibility of obtaining the MSSM [16].

The contributions to the scalar spectrum (45) from $j=y$ and $j=y+b_r$ give potential Higgs doublets with the correct hypercharges. We find that

$$n^{(33)}_{\mathrm{scalars}}(\mathbf{1},\mathbf{2}) = \sum_i \left[2(\delta_{d_i,\frac{1}{2}(a+3b)} + \delta_{d_i,-\frac{1}{2}(a+3b)}) + \delta_{d_i,\frac{1}{2}(3a+b)} + \delta_{d_i,-\frac{1}{2}(a+3b)}\right] \tag{59}$$

$$n^{(33)}_{\mathrm{scalars}}(\mathbf{1},\bar{\mathbf{2}}) = \sum_i \left[2(\delta_{d_i,\frac{1}{2}(3a+b)} + \delta_{d_i,-\frac{1}{2}(3a+b)}) + \delta_{d_i,\frac{1}{2}(5a-b)} + \delta_{d_i,-\frac{1}{2}(5a-b)}\right] \tag{60}$$

Comparing (59) and (60) with (57), we see that

$$n^{(33)}_{\mathrm{scalars}}(\mathbf{1},\mathbf{2}) + n^{(33)}_{\mathrm{scalars}}(\mathbf{1},\bar{\mathbf{2}}) \geq n^{(33)}_{\mathrm{fermions}}(\bar{\mathbf{3}},\mathbf{1}) - \delta_{3a+2b,0} \tag{61}$$

The $(37_r) + (7_r3)$-sector spectrum is calculated in a similar way, and leads to

$$\sum_{j=0}^{N}\Big[(\mathbf{n}_j,\bar{\mathbf{u}}^r_{j+\frac{1}{2}b_r})+(\mathbf{n}_j,\mathbf{u}^r_{-j-\frac{1}{2}b_r}) + (\bar{\mathbf{n}}_j,\mathbf{u}^r_{j-\frac{1}{2}b_r})+(\bar{\mathbf{n}}_j,\bar{\mathbf{u}}^r_{-j+\frac{1}{2}b_r})\Big] \text{ (fermions)} \quad (62)$$

$$\sum_{j=0}^{N}\Big[(\mathbf{n}_j,\bar{\mathbf{u}}^r_{j-\frac{1}{2}(b_s+b_t)}) + (\mathbf{n}_j,\mathbf{u}^r_{-j+\frac{1}{2}(b_s+b_t)})+(\bar{\mathbf{n}}_j,\mathbf{u}^r_{j+\frac{1}{2}(b_s+b_t)}) + \\ + (\bar{\mathbf{n}}_j,\bar{\mathbf{u}}^r_{-j-\frac{1}{2}(b_s+b_t)})\Big] \text{ (scalars)} \quad (63)$$

Then

$$n^{(37)}_{\text{fermions}}(\mathbf{3},\mathbf{1}) = \sum_{r=1,2} 2u^r_0 + u^3_{\frac{1}{2}(a-b)} + u^3_{-\frac{1}{2}(a-b)} \quad (64)$$

$$n^{(37)}_{\text{fermions}}(\bar{\mathbf{3}},\mathbf{1}) = \sum_{r=1,2} (u^r_{a+b} + u^r_{-(a+b)}) + u^3_{\frac{1}{2}(3a+b)} + u^3_{-\frac{1}{2}(3a+b)} \quad (65)$$

$$n^{(37)}_{\text{scalars}}(\mathbf{1},\mathbf{2}) = \sum_{r=1,2} (u^r_{a+b} + u^r_{-(a+b)}) + u^3_{\frac{1}{2}(a+3b)} + u^3_{-\frac{1}{2}(a+3b)} \quad (66)$$

$$n^{(37)}_{\text{scalars}}(\mathbf{1},\bar{\mathbf{2}}) = \sum_{r=1,2} (u^r_{2a} + u^r_{-(2a)}) + u^3_{\frac{1}{2}(3a+b)} + u^3_{-\frac{1}{2}(3a+b)} \quad (67)$$

Clearly we require that

$$u^1_0 = u^2_0 = u^3_{\frac{1}{2}(a-b)} = u^3_{-\frac{1}{2}(a-b)} = 0 \quad (68)$$

in order to exclude exclude unwanted $\bar{q}^c_L$ states. The definition (38) again ensures that these states have the correct standard-model weak hypercharge assignments. Comparing (66) and (67) with (65), we see that

$$n^{(37)}_{\text{scalars}}(\mathbf{1},\mathbf{2}) + n^{(37)}_{\text{scalars}}(\mathbf{1},\bar{\mathbf{2}}) \geq n^{(37)}_{\text{fermions}}(\bar{\mathbf{3}},\mathbf{1}) \quad (69)$$

Combining this with the analogous (33)-sector result (61), we conclude that the total number $n(H)+n(\bar{H})$ of Higgs doublets and the total number $n(q^c_L)$ of quark singlets satisfy

$$n(H) + n(\bar{H}) \geq n(q^c_L) - \delta_{3a+2b,0} \quad (70)$$

As before, any standard-like model requires that there are $n(q^c_L) = 6$ quark singlet states. Thus again the Higgs content cannot be that of the standard model, nor that of the MSSM.

We have shown that non-minimal Higgs content is unavoidable in models which seek to replicate the standard model spectrum, or that of the MSSM, by starting with D3-branes situated at a singular point of a $\mathbb{Z}_N$ orbifold or orientifold. This conclusion is independent of the order N of the point group $\mathbb{Z}_N$. It is also independent of the gauge groups living on the D7-branes that have to be introduced to complete the

spectrum, and which are in any case inescapable to ensure cancellation of the RR tadpoles. Of course, tadpole cancellation, or equivalently cancellation of the non-abelian anomalies, constrains these groups, and it may be that even stronger lower limits on the Higgs content can be obtained if these constraints are imposed. However, we have not explored this point further. As already noted, the orbifold models are known to be deficient in other respects, in particular their necessity for vector-like lepton-doublet matter. Orientifold models do not have this affliction, but to avoid vector-like quark-singlet matter we must take a non-supersymmetric embedding of the point group. Even so, vector-like "squark"-singlet matter *is* unavoidable. (We are using the term "squark" loosely here since there is no supersymmetry; we mean simply scalars tranforming as the $(\mathbf{3}, \mathbf{1})$ representation of $SU(3)_c \otimes SU(2)_L$.) The $j = x$ contribution to the last term of the scalar spectrum (45) includes 2 copies of $(\bar{\mathbf{3}} \times \bar{\mathbf{3}})_a = \mathbf{3}$, since for this term $2j = -(a+b) = -b_1 = -b_2$. However, we regard the appearance of vector-like squarks as a less serious defect than the appearance of vector-like fermions, because large masses for the former can be generated by strong radiative corrections [22], whereas fermion masses are protected by chiralty considerations. There is a further objection to the orbifold and orientifold models of the type that we are considering, which is that the (37)+(73)-sectors contains matter that transforms non-trivially with respect to both the standard model gauge group and the D7-brane gauge groups. Since there is no evidence for any gauge symmetry other than that of the standard model gauge group, if these models were to occur in nature, then it must be that the non-abelian D7-brane gauge groups are completely broken. At most a single surviving gauged $U(1)$ can survive, with other $U(1)$s surviving only as global symmetries after taking account of Green-Schwarz terms and the possibility of scalars in the (77)-sectors acquiring non-zero vacuum expectation values (VEVs) to remove unwanted $U(1)$ gauge groups. In principle, by a judicious choice of Wilson lines, it might be possible to arrange this. However, such symmetry breaking does not change the numbers u_j^r of standard-model representations that occur in the (37_r)+(7_r3)-sectors, and so will not change the numbers of quark singlets or Higgs doublets. This is because the D3-branes are situated at the origin, and the massless (37_r)+(7_r3)-sector states therefore have both ends of the string at the origin. Thus, the influence of the Wilson lines is not felt. We have already observed that this same feature also ensures that the problem with instanton-induced flavour changing neutral currents that afflicts the intersecting brane models does not arise in these models. However, it might also make the emergence of a realistic mass hierarchy difficult to achieve. As observed in the introduction, another objection to non-supersymmetric models is that cancellation of RR tadpoles does not guarantee cancellation of the NSNS tadpoles. Thus although the complex structure moduli are fixed by the point group symmetry in any (orbifold or) orientifold model, the stabilsation of the dilaton remains problematic, as in the (non-supersymmetric) intersecting brane models. In any case, the conclusion that *all* such models have non-minimal Higgs content means that after electroweak symmetry breaking they are all afflicted with tree-level flavour-changing neutral currents mediated by Higgs exchange. The severity of the experimental limits on these processes means that these models too

are effectively dead. The only escape from this conclusion that we can see is if the (77)-sector VEVs are large and effectively remove some of the Higgs doublets from the low-energy spectrum. Such VEVs cannot affect the number $n(q_L^c)$ of quark singlet states, because mass terms for them can only arise from VEVs for the electroweak Higgses in the (33)- and (37)-sectors. However, in the absence of supersymmetry, the calculation of the required VEVs entails the calculation of the ϕ^4 terms in the effective potential for the (77)-sector scalars, and we have not attempted this. Even if it is possible in principle, it seems unlikely that the survival of a single $H + \bar{H}$ pair would be generic.

A similar analysis can be made for the left-right symmetric and Pati-Salam models with gauge groups $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R$ and $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$ respectively. In the latter case, lepton number is the fourth "colour" and a single fermionic generation (including a right-chiral neutrino) is contained in the representations $(\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$. In the former $\mathbf{4} \in SU(4)_c$ is $\mathbf{3} + \mathbf{1} \in SU(3)_c$ and $\bar{\mathbf{4}} = \bar{\mathbf{3}} + \mathbf{1}$. In both cases the Higgs bosons are required to be in the $(\mathbf{1}, \mathbf{2}, \mathbf{2}) \in SU(n_c) \otimes SU(2)_L \otimes SU(2)_R$ where $n_c = 3$ or 4 is the number of colours. To get the required gauge group, instead of (10) we take

$$V_3 = \frac{1}{N}(x^{n_c}, y^2, z^2, d_1, d_2, ..., d_i, ...) \tag{71}$$

where x, y, z and d_i are all different integers (mod$N$). In the orbifold case we take the a_α as in (16). Then choosing $y = x + a$, as before, gives the required three copies of $(\mathbf{n_c}, \bar{\mathbf{2}}, \mathbf{1}) \in U(n_c) \otimes U(2)_L \otimes U(2)_R$. The right-chiral states transforming as $(\bar{\mathbf{n}}_c, \mathbf{1}, \mathbf{2})$ must all be in the 33 sector, and we get the required three copies when $z = x - a = y - 2a$. Then, unavoidably, there is non-minimal Higgs content since there are three copies of $(\mathbf{1}, \mathbf{2}, \bar{\mathbf{2}})$ in the bosonic sector. In the orientifold case, we take a_α as in (48). Taking $\tilde{V}_3$ to have the form (71), where $x = -\frac{1}{2}(a+b)$ and $y = \frac{1}{2}(a-b)$ as in (51), then gives the required $2(\mathbf{n_c}, \bar{\mathbf{2}}, \mathbf{1}) + (\mathbf{n_c}, \mathbf{2}, \mathbf{1})$ left-chiral matter content. The only way to get the required three copies of $(\bar{\mathbf{n}}_c, \mathbf{1}, \mathbf{2}) \in SU(n_c) \otimes SU(2)_L \otimes SU(2)_R$ then is if $z = \pm\frac{1}{2}(3a+b)$; the positive sign gives $2(\bar{\mathbf{n}}_c, \mathbf{1}, \bar{\mathbf{2}}) + (\bar{\mathbf{n}}_c, \mathbf{1}, \mathbf{2}) \in U(n_c) \otimes U(2)_L \otimes U(2)_R$, while the negative sign gives the conjugate $U(2)_R$ representations. As before, this fixes the Higgs boson content. In the former case we get $2(\mathbf{1}, \bar{\mathbf{2}}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}, \mathbf{2})$, while in the latter the conjugate $U(2)_R$ representations arise. Either way the Higgs content is again non-minimal, since we have more than one copy of one of the multiplets. It is also insufficient to give Yukawa couplings for all left- and right-chiral states.

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